CODE:-AG-2.1899


## General Instructions :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section - A comprises of 10 question of 1 mark each. Section - B comprises of 12 questions of 4 marks each and Section - C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section - A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 2 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 3 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

## सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड - अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड - ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड - स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 2 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 14 हैं।
7. प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।

## Pre-Board Examination 2010-11



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|  | least 4 successes. Ans $=\frac{496}{729}$ |
| :---: | :---: |
| Q. 7 | Show that $\left\|\begin{array}{lll}\boldsymbol{x}+1 & \boldsymbol{x}+2 & \boldsymbol{x}+\boldsymbol{a} \\ \boldsymbol{x}+2 & \boldsymbol{x}+3 & \boldsymbol{x}+\boldsymbol{b} \\ \boldsymbol{x}+3 & \boldsymbol{x}+4 & \boldsymbol{x}+\boldsymbol{c}\end{array}\right\|=0$ where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are in A.P. |
| Q. 8 | Evaluate: $\int_{0}^{a} \frac{1}{x+\sqrt{a^{2}-x^{2}}} d x$. Ans $\frac{\pi}{4}$ |
| Q. 9 | Find x , y if the points $(\mathrm{x},-1,3),(3, y, 1)$ and $(-1,11,9)$ are collinear. Ans : $\mathrm{x}=2, \mathrm{y}=-5$ |
| Q. 10 | Find the least value of a such that the function $x^{2}+a x+1$ is increasing on [1,2]. Ans a $=-2$ |
|  | Section B |
| Q. 11 | Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2 \hat{i}-3 \hat{j}+4 \hat{k}$. Also deduce its Cartesian form. Ans $r \cdot(2 i-3 j+4 k)=6, \text { cartesian }=2 x-3 y+4 z=6$ <br> OR <br> A variable plane which remains at a constant distance of 9 units from the origin, cuts the coordinate axes at the points A, B and C. Show that the locus of the centroid of $\triangle A B C$ is $\frac{1}{x^{2}}+\frac{1}{y^{2}}+\frac{1}{z^{2}}=\frac{1}{9}$. |
| Q. 12 | Evaluate : $\int \frac{1-x^{2}}{x(1-2 x)} d x$.Ans $\frac{1}{2} x+\log \|x\|-\frac{3}{4} \log \|1-2 x\|+c$. <br> OR <br> Evaluate: $\int_{-\pi}^{\pi} \frac{2 x(1+\sin x)}{1+\cos ^{2} x} d x$. Ans $=\pi^{2}$ |
| Q. 13 | In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses. Ans -91/54 <br> OR <br> The sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution. Ans $\mathrm{p}(\mathrm{x}=\mathrm{r})={ }^{32} c_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{32-r} \mathrm{r}=0,1,2, \ldots \ldots .32$ |
| Q. 14 | Using properties of determinant prove that $\left\|\begin{array}{lll}b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y\end{array}\right\|=2\left\|\begin{array}{lll}a & b & c \\ p & q & r \\ x & y & z\end{array}\right\|$. |
| Q. 15 | Find the intervals in which the function $f$ given by $f(x)=x^{3}+\frac{1}{x^{3}}, x \neq 0$ is (i) increasing (b) decreasing .\{Ans.(i) $f(x) \uparrow$ on $(-\infty,-1] \cup[1, \infty)$ <br> (ii) $f(x) i s \downarrow[-1,0) \cup(0,1]$ <br> OR |

Find all the tangents to the curve $y=\cos (x+y),-2 \pi \leq x \leq 2 \pi$ that are parallel to the line x $+2 y=0$. Ans $\left(\frac{\pi}{2}, 0\right) \&\left(\frac{-3 \pi}{2}, 0\right) \quad$ equation of tangent are

|  | $2 x+4 y-\pi=0 \& 2 x+4 y+3 \pi=0$ |
| :---: | :---: |
| Q. 16 | If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c}=\vec{b}$ and $\vec{a} \cdot \vec{c}=3$. Ans. $\frac{5}{3} i+\frac{2}{3} j+\frac{2}{3} k$ |
| Q. 17 | Evaluate: $\int \frac{x^{2}}{(x \sin x+\cos x)^{2}} d x$. Ans $\frac{-x}{\cos x(x \sin x+\cos x)}+\tan x$ |
| Q. 18 | Obtain the differential equation by eliminating $\mathbf{a}$ and $\mathbf{b}$ from the equation $y=e^{x}(a \cos x+b \sin x)$. |
| Q. 19 | Prove that the function $f: R-\{3\} \rightarrow R-\{1\}$ given by $f(x)=\frac{x-2}{x-3}$ is bijection. |
| Q. 20 | Find the equations of the line which intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \& \frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and passes through the point $(1,1,1)$. Ans. $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$ |
| Q. 21 | Solve the equations $\tan ^{-1} \frac{1-x}{1+x}=\frac{1}{2} \tan ^{-1} x,(x>0)$. Ans $x=\frac{1}{\sqrt{3}}$ |
| Q. 22 | If $\mathbf{x}=\mathbf{a} \sin \mathbf{p t}$ and $\mathbf{y}=\mathbf{b} \cos \mathbf{p t}$, find the value of $\frac{d^{2} y}{d x^{2}}$ at $\mathbf{t}=\mathbf{0}$. |
|  | Section C |
| Q. 23 | Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot(\hat{i}+2 \hat{j}+3 \hat{k})-4=0 \& \vec{r} \cdot(2 \hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r} .(5 \hat{i}+3 \hat{j}-6 \hat{k})+8=0$. Also the inclination of this plane with the xy plane. Ans : $r .(33 i+45 j+50 k)-4120 \lambda=\frac{7}{19} \cos \theta=\frac{50}{\sqrt{5614}}$ |
| Q. 24 | Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $8 / 27$ of the volume of the sphere. Ans: $f(x)=\frac{1}{3} \pi(r-x)(r+x)^{2} \therefore x=\frac{r}{3}$ <br> OR Show that the semi- vertical angle of a right circular cone of given surface area and maximum volume is $\sin ^{-1}\left(\frac{1}{3}\right)$. Ans: $\mathrm{S}=\pi r^{2}+\pi r l \Rightarrow l=\frac{s-\pi r^{2}}{\pi r} \& \mathrm{f}(\mathrm{r})$ $=V=\frac{1}{3} \pi r^{2} h \Rightarrow V^{2}=\frac{S^{2} r^{2}}{9}-\frac{2 s \pi r^{4}}{9} \therefore s=4 \pi r^{2}$ |
| Q. 25 | Sketch the graph of $f(x)=\left\{\begin{array}{cc}\|x-2\|+2, & x \leq 2 \\ x^{2}-2, & x>2\end{array}\right.$.Evaluate $\int_{0}^{4} f(x) d x$. What does the value this integral represent on the graph? Ans $=\frac{62}{3}$ <br> OR <br> Find the area of the region in the first quadrant enclosed by the line $\mathbf{y}=\mathbf{x}$ and the circle $\mathbf{x}^{2}+$ $\mathbf{y}^{2}=32$ above $\mathbf{x}$ axis.. Ans. $=4 \pi u_{n i t}{ }^{2}$ |
| Q. 26 | State the condition under which the following system of equations have a unique solutions. If |

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|  | $\mathrm{A}=\left[\begin{array}{ccc}9 & 7 & 3 \\ 5 & -1 & 4 \\ 6 & 8 & 2\end{array}\right]$,find $A^{-1}$ and hence solve the following system of $y+4 z=1 ; 6 x+8 y+2 z=4$. <br> Ans. $A^{-1}=\frac{-1}{70}\left[\begin{array}{ccc}-34 & 10 & 31 \\ 14 & 0 & -21 \\ -46 & -30 & -44\end{array}\right], x=1, y=0, z=-1$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Q. 27 | A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines are as follows: |  |  |  |
|  | Machine | Area occupied | Labour force | Daily output(in units) |
|  | A | $1000 \mathrm{~m}^{2}$ | 12 men | 60 |
|  | B | $1200 \mathrm{~m}^{2}$ | 8 men | 40 |
|  | He has maximum area of $9000 \mathrm{~m}^{2}$ available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximize the daily output? |  |  |  |
| Q. 28 | Suppose a girl throws a die. If she gets a 5 or 6 , she tosses a coin three times and note the number of heads. If she gets a $1,2,3$ or 4 , she tosses a coin once and notes whether a heads or tail is obtained. If she obtained exactly one head ;what is the probability that she threw $1,2,3$ or 4 with the die . <br> OR <br> Let the number of times a candidate applies for a job be X and $\mathrm{P}(\mathrm{X}=\mathrm{x})$ denotes the probability that he will be selected $x$ times. Given that $P(X=x)=\begin{array}{ll} (k+1) x & \text { if } x=0 \\ 2 k x & \text { if } x=1 \text { or } 2 \\ k(6-x), & \text { if } x=3 \text { or } 4 \text { or } 5 \end{array} \quad \text { where } k \text { is } a+v e \text { real number. }$ <br> (a) Find the value of k.(b) What is the probability that he will be selected exactly three times.(c) What is the probability that he will be selected at least once.(d) Find the mean and variance of the probability distribution of X. Ans $(a) k=\frac{1}{12}(b)=\frac{1}{4}(c)=1(d)$ mean $=\frac{8}{3}$, var iance $=\frac{25}{18}$ |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| Q. 29 | Using limits of sum find the integral of $\int_{1}^{3}\left(3 x^{2}+e^{2 x}\right) d x$. Ans. $26+\frac{e^{6}-e^{2}}{2}$ |  |  |  |
|  | -__ $\mathrm{X}_{-}$ |  |  |  |

